SOL HW 6.1

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Name:________________________ Date:________________________

Math 10 Honours: HW Section 6.1 Permutations and FCP

1. Simplify each of the following factorial expressions. Show all your work and steps:

a) $\frac{10!7!4!}{9!6!3!}$ $\frac{10\times\cancel{9!}\times7\times\cancel{6!}\times4\times\cancel{3!}}{\cancel{9!}\times\cancel{6!}\times\cancel{3!}}$ $=10\times7\times4$ $=280$	b) $\frac{(99!)(101!)}{(98!)(102!)}$ $\frac{99\times\cancel{98!}\times\cancel{101!}}{\cancel{98!}\times102\times\cancel{101!}}$ $=\frac{99}{102}=\frac{33}{34}$	c) $\frac{7!-6!-5!}{6!-5!}$ $\frac{\cancel{5!}(7\times6-6-1)}{\cancel{5!}(6-1)}$ $=\frac{42-7}{5}=\frac{35}{5}=7$
d) $\frac{10!\times9!\times8!}{(9!)^2}$ $\frac{10\times\cancel{9!}\times\cancel{9!}\times8!}{\cancel{9!}\times\cancel{9!}}$ $=40,320\times10$ $=403,200$	e) $\frac{100!-99!}{99!-98!}$ $\frac{\cancel{99!}(100-1)}{\cancel{98!}(99-1)}$ $=\frac{99(99)}{98}=\frac{9801}{98}$	f) $\frac{7!\times5!}{10!}\left(\frac{9!}{3!\times5!}-\frac{10!}{2!\times7!}\right)$ $\frac{7!\times\cancel{5!}\times\cancel{9!}}{{}_{10}\cancel{10!}\quad 2!\,\cancel{5!}}\left(\frac{1}{3}-\frac{10}{7}\right)$ $\frac{7!}{10\times2}\left(\frac{7-30}{21}\right)=\frac{7\cdot6\cdot5\cdot4\cdot3}{2\cdot5}\left(\frac{-23}{7\cdot3}\right)$ $=-276$
g) $\frac{(n-1)!(n+1)!}{n!}$ $\frac{(n-1)!\,(n+1)\cancel{(n!)}}{\cancel{(n!)}}$ $(n-1)!\,(n+1)$	h) $\frac{n!-(n-1)!}{(n+1)!-2(n-1)!}$ $\frac{n\cancel{(n-1)!}-\cancel{(n-1)!}}{(n+1)(n)\cancel{(n-1)!}-2\cancel{(n-1)!}}$ $\frac{n-1}{n(n+1)-2}=\frac{(n-1)}{(n+2)(n-1)}$ $=\frac{1}{n+2}$	i) $\frac{(n-2)!}{(n-1)!}-\frac{(n-3)!}{(n-1)!}$ $\frac{1\cancel{(n-2)!}}{(n-1)\cancel{(n-2)!}}-\frac{\cancel{(n-3)!}\cdot1}{(n-1)(n-2)\cancel{(n-3)!}}$ $\frac{1}{n-1}-\frac{1}{(n-1)(n-2)}$ $\frac{(n-2)-1}{(n-1)(n-2)}=\frac{n-3}{(n-1)(n-2)}$

2. Solve each equation for "n"

a) $nP_2=20$ $\frac{n!}{(n-2)!}=20$ $n(n-1)=20$ $n^2-n-20=0$ $(n-5)(n+4)=0$ $\boxed{n=5}$ or Guess & Check $5\times4=20$ $\boxed{n=5}$	b) $nP_3=4080$ $\frac{n!}{(n-3)!}=4080$ $(n)(n-1)(n-2)=4080$ ⋮ Just use guess chk. $17\times16\times15=4080$ $\boxed{n=17}$	c) $6P_n=120$ $6\times5\times4=120$ $\boxed{n=3}$

3. In how many different ways can three students be seated in a row of six desks?

$6P_3=6\times5\times4$

$=120$

4. In how many different ways can first, second, and third prize winners be chosen in a random drawing if 40 people entered a contest?

$40P_3=40\times39\times38$

$=59,280$

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5. In how many different ways can the letters in the word “DANGER” be scrambled?

$6! = 720.$

6. Tom has 9 sweaters, 7 jeans, and 5 pairs of shoes. How many different outfits consisting of a sweater, jeans, and a pair of shoes can Tom chose?

$9 \times 7 \times 5$
$= 315$ //.

7. A student, taking a True-False test, randomly guesses all 10 questions. How many different sets of answers could be produced?

$2^{10} = 1024$ //

8. Four candidates are running for president of an organization. Their names are placed in a ballot of random order. How many different ways are there to order the names?

$4! = 24$ //

9. What is the sum of all possible odd four digit numbers that can be formed using digits 2, $\boxed{3}$, 4, and $\boxed{5}$? Each digit can only be used once.

$3 \times 2 \times 1 \; \boxed{2}$

$= 12$ numbers //.

① units digit: $6(3) + 6(5) = 48$

② Tens digit: $10 \times [4(2) + 4(4) + 2(3) + 2(5)]$
$= 10[24 + 16] = 400$

③ Hundreds digit $= 100(40) = 4000$

④ Thousands: $= 1000(40) = 40{,}000$.

44,448 //.

10. If the digits 1, 2, 3, 5, and 7 can be used more than once, how many different even three-digit numbers can be written?

$4 \times 3 \times \boxed{1} = 12$.

This must be a '2'.

11. Using each of the digits 2, 5, 7, and 8 once in each number, how many different four-digit numbers can be formed? How many of these numbers are even?

① $4 \times 3 \times 2 \times 1 = 24$ numbers

② $3 \times 2 \times 1 \times 2 = 12$ numbers are even.

12. How many four letter words begin and end with the same letter?

$\underline{26} \times \underline{26} \times \underline{26} \times \underline{1} = 26^3.$

• IT DOESN'T NECESSARILY HAVE TO BE A WORD

13. A restaurant has four types of beverages and six types of sandwiches. How many different orders consisting of one beverage and two sandwich are there?

$4 \times 6 = 24$ //

14. A cafeteria has 3 different soups, 4 different main course, and 5 different desserts. How many different meals can Jim have if he must order something and he can not buy the same thing twice?

SOUPS: A, B, C. $2 \times 2 \times 2 = 8$

MAINS: D E F G $2 \times 2 \times 2 \times 2 = 16$

DESSERT: 32

$\rightarrow 8 \times 16 \times 32 - 1 = 4095$ //

SUBTRACT THE CASE WHEN HE ORDERS NOTHING

15. Jazmin wants to make a sandwich. There are 3 types of bread to choose from: white, wheat, or all grain. She can have one, two, or all three types of meat: ham, turkey, and chicken. She can have one of the two condiments or none: Mayo and mustard. How many different sandwiches can she make?

BREAD: 3 $\times$ HAM: $2^3 - 1$ $\times$ SAUCE: 3 $= 63$.

16. Jack has twice as many shirts as pants. What is the fewest number of shirts he needs in order to wear a different combination of shirt and pants each day of the year?

Pants: (n) Shirts: (2n)

$(2n)(n) > 365$

$n^2 > 182.5$

$\boxed{n > 13.5}$

HE NEEDS 28 SHIRTS

17. A 3-number combination lock contains numbers from 0 to 20. How many combinations are possible if the second and first digits are different, AND the second and third must also be different?

0 to 20: 21 options.

$$\boxed{21 \times \underbrace{20}_{\text{MUST ONLY BE DIFF FROM } 1^{st}} \times \underbrace{20}_{\text{MUST ONLY BE DIFF. FROM } 2^{nd}} = 8400}$$ //

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18. How many ways can Amy, Betty, Cindy, Donna, and Elaine sit in a row if Amy and Betty can not sit next to each other?

$5! - 2(4!)$

$120 - 2(24)$

$120 - 48 = 72$

19. 6 dating couples are sitting a row of 12 chairs. If each couple must sit next to each other, how many different arrangements can there be?

$6! \times 2^6$

$= 720 \times 64$

$= 46{,}080$

20. How many ways can 4 boys and 4 girls sit in a row if boys and girls are to sit in alternate seats?

• B_1 B_2 B_3 B_4 • G_1 G_2 G_3 G_4.

_ G _ G _ G _ G _ $\Rightarrow 4! \times {}_5P_4$

$= 4! \times 5 \times 4 \times 3 \times 2$

$= 576$

21. ~~Challenge~~: How many ways can 3 boys and 5 girls sit in a row if the boys can not sit next to each other?

_ G _ G _ G _ G _ G _ $\Rightarrow 5! \times {}_6P_3$

$= 120 \times 6 \times 5 \times 4$

$= 14{,}400$

22. In how many distinct ways can five children be seated around a circular merry-go-round which has five identical seats?

$\frac{5!}{5} = 4! = 24$